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A variational formulation of the thermo-mechanical problem for general dissipative composites

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Abstract

This study devises a method of two-scale analysis for a thermo-mechanically coupled problem of general dissipative composites in a variationally consistent manner. To ensure variational consistency, the formulation is based on an incremental variational framework, in which the instantaneous equilibrium state of the continuum body is determined by an inf-sup problem of a thermo-mechanically coupled rate potential. To address the difficulties arising from the nonlinearity of the transient heat conduction equation, the inf-sup problem for the heterogeneous solids is rewritten to involve logarithmic temperature as a state variable, and the multiplicative decomposition of heterogeneous temperature field is defined using macroscopic and fluctuation temperature fields. This setup makes the formulation of two-scale boundary value problems straightforward and ensures that the resulting governing equations on both scales are variationally consistent. In this study, the target structures are heterogeneous solids constructed from periodic alignments of infinitesimal or finite-sized unit cells. For the latter type of structure, we consider the transient changes in the microscopic temperature field and follow the formulation for a second-order computational homogenization scheme. Specifically, the stationary conditions corresponding to the inf-sup problem for the heterogeneous solids are approximately decomposed into micro- and macroscopic counterparts. The two-scale BVP is completed by introducing admissible and periodic boundary conditions for displacement and logarithmic temperature fluctuations. FE2-type semi-implicit numerical scheme are also constructed by extending the tangential homogenization technique to a thermo-mechanically coupled problem. Several numerical examples, such as heat conduction, thermoelastic, viscoplastic problems, are solved to verify the validity of our proposed method in terms of accuracy and convergence. We also study the effect of the unit cell size and the nonlinearity of the heat conduction phenomenon on multiscale coupled behaviors.

Keywords:

Thermo-mechanics Two-scale analysis Variationally consistent