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Iterative homogenization of quasiperiodic materials based on substitution rules

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Extended Abstract

Abstract

INTRODUCTION Recent technological advances in additive manufacturing allow the fabrication of innovative architectured materials, whose complex geometries yield interesting effective properties at the macroscopic scale. The determination of these effective properties as a function of the geometry is a key step in putting these architectured materials to use. Classically, architectured materials consist of either periodic geometries, completely determined by a single repeating unit cell and often leading to anisotropic effective behavior, or random geometries which usually display isotropic behavior. For these configurations, efficient methods are available that calculate their effective properties. Current research is looking to explore geometrical configurations between the restrictive frame of strict periodicity and the stochastic frame of random media. One example of such geometries are quasiperiodic materials, where the absence of the unit cell invalidates the efficient methods of periodic homogenization. This contribution proposes a method that exploits the underlying order present in quasiperiodic materials in order to efficiently compute their effective linear elastic behavior. METHODOLOGY One way of generating quasiperiodic geometries is a process called substitution. This iterative procedure creates the geometry by replacing each tile with a fixed arrangement of smaller tiles, called its image under the substitution rule. Each iteration of this substitution procedure is called an order. We propose a numerical method computing the effective elastic behavior of a quasiperiodic material based on the images of the substitution rule. Since the outlines of the original tiles are preserved under the substitution rule, the numerically obtained effective properties can be associated to each tile. This allows the effective stiffness tensors from the previous iteration to be reused for the next numerical homogenization. After an initialization step using as input the stiffness tensors of the studied quasiperiodic material, consecutive iterations are performed until convergence is achieved. A substitution order can be chosen independently in each step. FINDINGS & CONCLUSIONS The proposed method is applied to a selection of common quasiperiodic materials. Based on these examples it is shown that the order chosen for the initialization step is crucial to the efficiency of the method. The convergence is then reached for a very low number of consecutive iterations, regardless of their chosen order.. The proposed method is presented in the context of linear elasticity but its formalism can be extended to any linear physical property to be homogenized.

Keywords:

Homogenization Quasiperiodicity Architectured Materials Substitution