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Cut & Project homogenization and optimization of quasi-periodic architectured materials

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Extended Abstract

Abstract

Architectured materials are defined by a matter organisation at an intermediate scale between the macro-scale (the specimen) and the micro-scale (the constitutive material). It is then possible to define arbitrarily the intermediate-scale arrangement of the architectured material. When this arrangement corresponds to that of quasi-crystal structure the architectured material is then referred to as "quasi-periodic material". Historically quasi-crystals have been discovered, described and studied by crystallographs. Quasi-periodic materials are in between periodic and random materials and combine the advantages of both media. Among all quasi-periodic materials of dimension n, there is a subset that can be generated by projecting the lattice of a space of dimension m > n (the hyperspace) onto a hyperplane of dimension n <= 3 (the physical space). The definition of a coordinate system within the m dimensional affine space allow us to parametrize the position and the orientation of this hyperplane with a point and angles. If the tangents of these angles have irrational values, then the resulting medium is quasi-periodic, otherwise it is periodic. This construction is called “Cut \& Project”: 1) cut by the physical space; 2) projection onto this physical space. It is possible to set up an equivalent construction by swapping these two steps. From the hyperlattice, two coverings of the space by polytopes are trivial, the Voronoï set and its dual, the Delaunay set. The projection of the n-faces of the Voronoï set onto physical space and of the (m-n)-faces of the Delaunay set onto its orthogonal complement yields a periodic covering of hyperspace called the Klotz's tiling. Using this Klotz construction, the whole material can be defined on an m-torus and is therefore bounded. This torus corresponds to the primitive cell of the material described in hyperspace. The intersection of this hyper-primitive cell - taking into account periodicity - with physical space yields a quasi-periodic tiling of physical space. The aim of the present work is to determine the effective linear elastic properties of quasi-periodic media taking advantage of this hyperspace framework. Indeed the quasi-periodic material is completely defined by a periodic configuration in hyperspace. The homogenization of periodic media, using two-scale asymptotic development, yields exact results. Consequently the homogenization can be performed on the torus to determine the effective properties of the quasi-periodic material. This homogenization is presented by performing the Klotz construction from an hyperlattice in dimension m = 2 and a physical space with n = 1 resulting in a 1D quasi-periodic two-phase material. Using such low dimensions has the advantage not only of offering an easily understandable hyperspace, but also of providing an analytical reference solution for the effective Young's modulus of the 1D quasi-periodic material. The results performed by a dedicated in-house finite element method (FEM) code present excellent convergence of the effective properties towards analytical solution. Since the effective elasticity tensor's space can be parametrized it is possible to set up an optimization process to reach desired mechanical behavior.

Keywords:

architectured materials quasiperiodic homogeneization optimization cut and project