

A unified equivalent layer-wise theory for hygro-thermo-magneto-electro-elastic analysis of anisotropic doubly-curved shells of arbitrary shape

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Abstract

The contribution presents an advanced unified higher-order theoretical framework for the coupled hygro-thermo-magneto-electro-elastic analysis of laminated anisotropic doubly-curved shells of arbitrary geometry. The formulation is developed according to the Equivalent Layer-Wise (ELW) approach in principal coordinates, where the unknown field variables are expanded through the thickness using generalized thickness functions [1], based on power functions and Jacobi polynomials, enriched by a zigzag function, to accurately capture interlayer interactions. The governing equations are derived from the Master Balance Principle under thermodynamic equilibrium conditions, and account for the coupling among mechanical elasticity, electrostatics, magnetostatics, thermal conduction, and mass diffusion [2]. Each layer of the laminate is assumed to consist of multiphase materials with coupled physical properties. The anisotropic constitutive equations are obtained through an analytical Mori–Tanaka homogenization scheme, relying on the evaluation of the Eshelby tensor for an ellipsoidal inclusion embedded in an infinite matrix, thereby enabling accurate prediction of multifield interactions. Furthermore, a NURBS-based mapping technique is adopted to handle geometries of arbitrary shape. The resulting system of equations is solved numerically using the Finite Element Method (FEM), with both Lagrange- and Hermite-polynomial-based shape functions implemented in quadrilateral and triangular elements [3]. In the post-processing

stage, an efficient patch-based recovery procedure [4] based on Generalized Differential Quadrature (GDQ) is used to reconstruct the full three-dimensional (3D) response of the doubly-curved solid with high accuracy, satisfying the complete 3D multifield balance equations. A wide set of numerical examples is presented, including systematic comparisons with 3D FEM reference solutions and two-dimensional Navier-type analytical results for validation [5]. In addition, extensive parametric studies are carried out to investigate the effect of multifield coupling on the global structural response of straight, singly-curved, and doubly-curved shells with different lamination schemes, including configurations with a central softcore [6]. Particular attention is devoted to hygro-thermal effects, magneto-electro-elastic coupling, pyroelectric and magnetoelectric phenomena, as well as classical piezoelectric and piezomagnetic interactions. Furthermore, the variation in the material properties distribution is modelled to investigate its effect on the structural response, considering polynomial and trigonometric material distributions. Finally, selected applications related to the mechanical response of laminated anisotropic doubly-curved shells are also shown, related to structures made of advanced materials including Carbon Nanotubes (CNTs) reinforcements, Functionally Graded Materials (FGMs), and honeycomb cells. Overall, the proposed model provides a comprehensive and computationally efficient tool for the analysis of advanced structural components where strong multifield coupling effects play a critical role.

Keywords:

Anisotropic materials. Doubly-curved shells. Higher-order theories. Finite element method. Generalized differential quadrature. Multifield analysis. Unified formulation.

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