Modelling pore closure in slowly degrading poroelastic materials driven by fluid flow

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BACKGROUND & PURPOSE

Hydrogels and other poroelastic materials can weaken upon interaction with chemical species, such as enzymes [1] dissolved within the interstitial fluid or simply by the solvent itself [2]. Several medical devices, such as ureteric stents [3] and porous polymer scaffolds [4], exploit degradability in their designs. Each application benefits from an understanding of the precise mechanism and timescale of degradation of these materials. In this work, we combine ideas from three existing frameworks [5–7] to create a mathematical model of a poroelastic material weakening due to a solute transported through the system via pressure-driven flow. In flow-driven compression of poroelastic media, it has been shown that a steady state of the system can be reached provided a critical pressure drop is not exceeded [5]. We investigate what happens to this steady state when the material stiffness slowly weakens.

MODELLING

We consider flow-driven uniaxial compression of a poroelastic material of initial length L with a free, permeable boundary on the left and a fixed, permeable boundary on the right of the domain. Our governing system consists of a nonlinear advection-diffusion equation for the porosity ϕ_f, a decay equation for the Young's modulus E, and an advection-diffusion equation for the solute concentration c. The rate of change in E is proportional to $-c(E - E_{\min})$, where $E_{\min}$ is the minimum Young's modulus of the material.

INITIAL AND BOUNDARY CONDITIONS

We suppose that the porosity and Young's modulus are spatially uniform initially, with values of $\phi_{f,0}$ and E_0, respectively, and that there is no solute in the domain. We impose a fixed pressure drop Δp across the domain and assume that there is no effective elastic stress on the left boundary. We also impose kinematic conditions on both boundaries to close the problem.

NUMERICAL RESULTS

Using a parameter set inspired by porous polymer scaffolds used in tissue engineering [4], we numerically solve the model using the finite element package FEniCS [8] and plot ϕ_f against the Lagrangian spatial coordinate, X. The weakening timescale t_E is much longer than that of poroelastic relaxation (t_ϕ) for this parameter set. The evolution of the system is split into two phases: initial poroelastic relaxation followed by quasi-steady weakening-driven deformation.

In some cases, the porosity on the right of the domain, $\phi_{f,r}$, reaches zero at a finite time, corresponding to pore closure (see Fig. 1a).

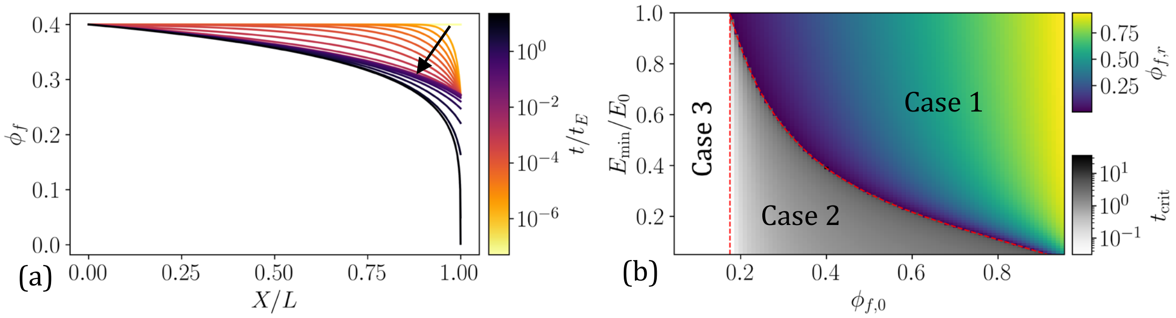

Fig. 1. (a) Spatial porosity profiles at given times t, measured relative to t_E. (b) Steady state porosity on the right boundary (Case 1) and time of pore closure (Case 2) in $(E_{\min}, \phi_{f,0})$ parameter space. In Case 3, t_{crit} is equal to zero.

QUASI-STEADY STATE ANALYSIS

To further investigate pore closure, we analytically calculate leading-order contributions to ϕ_f, E and c, treating the ratio t_ϕ/t_E as a small parameter. We find that the long-term qualitative behaviour of the system is governed by a single dimensionless parameter, $\alpha_{\min}$, which depends on the pressure drop, initial porosity, Poisson's ratio and minimum Young's modulus. We fix Δp and investigate the effect of varying $E_{\min}$ and $\phi_{f,0}$. In Case 1, if $E_{\min}$ and $\phi_{f,0}$ are large enough, the system reaches a steady state. We plot the steady-state value of $\phi_{f,r}$ for Case 1 in Fig. 1b. In Case 2, the porosity drops to zero on the right at a finite time t_{crit}; we plot the analytical value of t_{crit} for different values of $E_{\min}$ and $\phi_{f,0}$ in the Case 2 segment of Fig. 1b. Near the boundary curve between Cases 1 and 2, small changes in $E_{\min}$ can cause large changes in $\phi_{f,r}$ (Fig. 1b), which demonstrates that the qualitative behaviour of the system is sensitive to $E_{\min}$. In Case 3, if the initial porosity is too small for a given pressure drop, the pores close immediately on the right boundary, and there is no solution to the model.

CONCLUSIONS

We have developed a mathematical model for a one-dimensional poroelastic material weakening in the presence of a dissolved solute. We have found that when we impose a fixed pressure drop across the domain, there are three different qualitative cases describing the long-term behaviour of the system, assuming that the timescale of weakening is long compared to that of poroelastic relaxation. Crucially, weakening the material can cause the pores to close at a finite time determined by the material parameters and the pressure drop.

REFERENCES

[1] T. K. L. Meyvis *et al.* 1999 *Journal of Rheology* **43**, 933–950.
[2] B. Ozcelik 2016 *Degradable hydrogel systems for biomedical applications*, chap. 7, 173–188.
[3] A. A. Barros *et al.* 2016 *Applied Materials Today* **5**, 9–18.
[4] C. E. LeBlon *et al.* 2013 *Journal of Applied Polymer Science* **128**, 2701–2712.
[5] C. W. MacMinn *et al.* 2016 *Physical Review Applied* **5**, 044020.
[6] R. Abi-Akl *et al.* 2019 *Soft Matter* **15**, 4098.
[7] M. Fiori *et al.* 2025 *Journal of Fluid Mechanics* **1009**, A15.
[8] M. S. Alnaes *et al.* 2015 *Archive of Numerical Software* **3**, 9–23.

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