Minimal Parameter Fitting of Incompressible Visco-Hyperelastic Models via Automatic Differentiation

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INTRODUCTION

The mechanical characterization of soft materials is a long researched but still challenging field. In commercial finite element (FE) solvers there are a great variety of constitutive models available that can characterize complex material behaviour, including finite strain deformations with inelastic properties (e.g. viscoelasticity, Mullins-effect, permanent deformations, etc.). Yet, due to the highly nonlinear behaviour of such constitutive models, identifying material parameters is a challenging task. A commonly applied method is to perform various measurements with load cases specifically engineered to characterize the different deformation mechanisms separately. This method, however, may result in highly inaccurate stress solutions for complex load cases, where all deformation mechanisms are presented simultaneously. On the other hand, an FE-based method can provide accurate fitting even for the most complex load cases, but with highly increased fitting time and computational costs. In this contribution, we present a differentiable incompressible visco-hyperelastic constitutive model that allows efficient parameter identification from measurement data obtained from complex cases. The proposed numerical parameter fitting method not only ensures high accuracy, fast parameter convergence for even large number of parameters, but also allows for the estimation of parameter uncertainty and sensitivity analysis.

METHODS AND RESULTS

In this talk we focus on visco-hyperelastic material behaviour of incompressible (rubber-like) solids. We adopt the generalized standard solid constitutive model, where the nonlinear elastic behaviour is characterized by an incompressible hyperelastic model, while the rate-dependent behaviour is described using Prony-series.

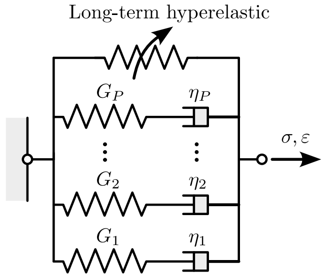

Figure 1. The schematics of the investigated finite-strain visco-hyperelastic standard-solid model

Due to finite strain deformations the constitutive model should be obtained using the deviatoric split, namely

$$\boldsymbol{\tau}^D(t) = \boldsymbol{\tau}_0^D(t) + \operatorname{dev}\int_0^t \frac{\dot{G}(s)}{G_0}\overline{\boldsymbol{F}}_t^{-1}(t-s)\boldsymbol{\tau}_0^D(t-s)\overline{\boldsymbol{F}}_t^{-T}(t-s)\mathrm{d}s, \tag{1}$$

$$\boldsymbol{\tau}^H(t) = \boldsymbol{\tau}_0^H(t) + \int_0^t \frac{\dot{K}(s)}{K_0}\boldsymbol{\tau}_0^H(t-s)\mathrm{d}s. \tag{2}$$

where $\boldsymbol{\tau_0}$ stands for the instantaneous hyperelastic Kirchhoff-stress solution. The numerical stress solution can be expressed using the following numerical integrations scheme

$$\boldsymbol{\tau}(t+\Delta t) = \left(1-\sum_{k=1}^{P} a_k\, g_k\right)\boldsymbol{\tau}_0^D(t+\Delta t) - \sum_{k=1}^{P} b_k\, g_k \hat{\boldsymbol{\tau}}_0^D(t) - \sum_{k=1}^{P} c_k \hat{\boldsymbol{\tau}}_k^D(t) + p\boldsymbol{I}, \tag{3}$$

where a_k, b_k, c_k are integration constants that depend on the g_i, τ_i Prony parameters and Δt. During the fitting process we assume that the data is obtained from measurements with homogenous loading, and we find the least mean square estimators of the material model parameters by minimizing

$$\widehat{\boldsymbol{p}} = \underset{\boldsymbol{p}}{\operatorname{argmin}} \frac{1}{2}\sum_{i=1}^{N}\left(P_{\mathrm{meas}}(t_i) - P_{\mathrm{model}}(t_i, \boldsymbol{p})\right)^2 \tag{4}$$

subject to all constitutive constraints ($\sum g_i < 1$; $\mu_0 > 0$; $\tau_i > 0$; $0 < g_i < 1$). The parameter vector $\boldsymbol{p}$ collects the Prony parameters (g_i, τ_i), with $i = 1 \ldots P$ and the hyperelastic parameters $p_{H,j}$. Using the differentiable implementation of the constitutive model, both the gradients and the Hessian of the mean square error are available, that are required for efficient optimisation and the computation of parameter uncertainty. Furthermore, we select minimal models via methods from statistics and data-science, including hypothesis testing and information criteria, and compare the results from the different approaches.

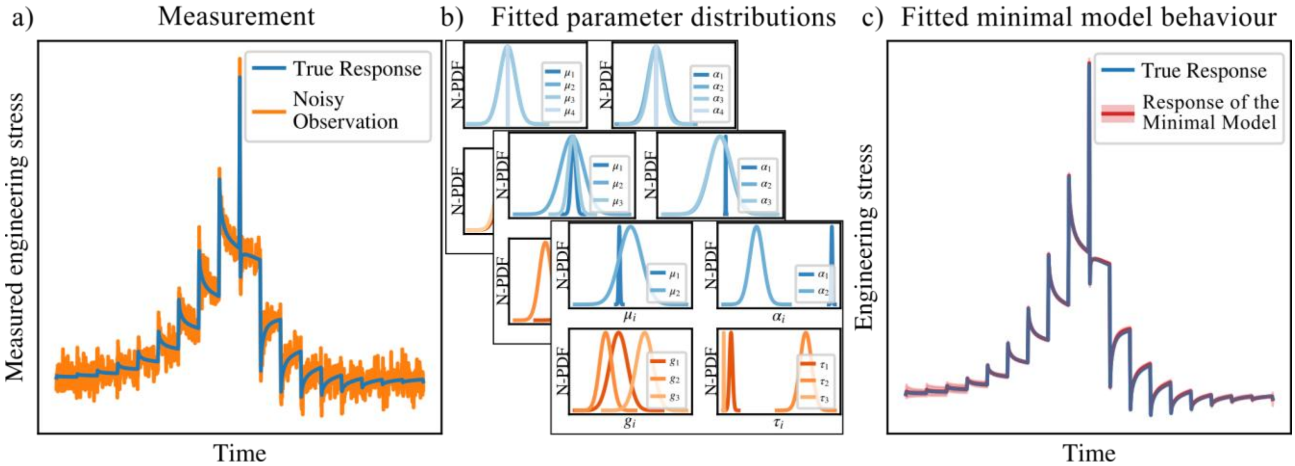

Figure 2. Sketch of the model selection process: a) Measured data b) Normalised probability density functions (N-PDF) of the fitted model parameters c) Result of the fitted model with 6-sigma uncertainty regions

REFERENCES

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