Predicting safety limits for Particle-Reinforced Metal Matrix Composites (PRMMCs) via multiscale shakedown analysis

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1 Extended abstract

Due to the high strength-to-weight ratio and excellent mechanical properties, Particle-Reinforced Metal Matrix Composites (PRMMCs) are increasingly used in aerospace, automotive, and civil engineering applications. However, predicting their mechanical behavior under cyclic and variable loads remains unexplored, especially for applications where failure states must be prevented in order to achieve extended life service. These type of advanced materials consist of a metallic matrix that is reinforced to enhance its properties; ceramic, alumina or boron carbide particles are typically used.

Lower-bound shakedown analysis, based on Melan's shakedown theorem [1], has been proposed as a powerful framework for evaluating the long-term response of materials subjected to extreme loading scenarios, including cyclic and time-independent loading paths. This theorem establishes that shakedown under repeated or cyclic loading occurs if a self-equilibrated residual stress (ρ_{ij}) exists, that doesn't depend on time. When this residual stress is superimposed on the purely elastic stress field associated with the applied loads (σ^e_{ij}), the resulting combined stress state must remain within the material's yield surface at every point on the structure and at every moment during the loading history. If such a residual stress exists, the material will no longer accumulate additional plastic deformation, thus leading the system to reach a shakedown state. The lower-bound shakedown theorem can be described mathematically as follows:

$$\forall \mathbf{x},\, \forall t : F\left(\alpha\, \boldsymbol{\sigma}^E(\mathbf{x},t) \,+\, \boldsymbol{\rho}(\mathbf{x})\right) \leq 0, \tag{1}$$

where $\alpha \geq 0$ is a dimensionless factor, and F is the yield criterion. This leads to an optimization problem that is constructed on the basis of Melan's theorem:

$$\begin{gathered}\alpha_{SD} = \max \alpha \\ \sum_{r=1}^{NG} \mathbb{C}_r \cdot \boldsymbol{\rho}_r = \mathbf{0} \\ \forall r \in [1, NG], \forall t \,:\, F\left(\alpha\, \boldsymbol{\sigma}_r^E(t) \,+\, \boldsymbol{\rho}\right) \leq 0,\end{gathered}$$

where NG is the total of Gaussian points (r) of the system, since Eq.1 is converted into a system of linear equations, the residual stress ($\boldsymbol{\rho}_r$) is now evaluated in the Gaussian points. Then, the equilibrium matrices $\mathbb{C}_r \in \mathbb{R}^{m_E \times 6}$ only depend on the system geometry and the chosen element type. One way to calculate them is via the principle of virtual work. Their

dimension is defined as $m_E = 3NK - NBC$, where NK is the total number of nodes and NBC is the number of kinematical boundary conditions.

Its application to PRMMCs remains limited. Conventional modeling approaches often idealize the microstructure and neglect the inherent heterogeneity of these composites; as a result, when the local interactions between reinforcement particles and the surrounding matrix are overlooked, the predictions may become inaccurate. Recent research [2, 3] has begun to address these shortcomings by the integration of microstructural features and boundary effects, paving the way for more reliable shakedown evaluations of PRMMCs.

A multiscale computational framework for interior-point shakedown analysis of PRMMCs developed as an extension of the previous methodology based on Melan's theorem [4, 5] is presented. Realistic 3D micro-structures are generated using an established technique, capturing particle morphology, size, and spatial distribution. Homogenization techniques are then applied to extract the effective material properties from the representative volume elements, which are later used as input for the macroscopic shakedown analysis. This framework enables a systematic investigation of how key micro-structural characteristics, like the particle size, the volume fraction, and the boundary conditions influence the shakedown limits.

By integrating realistic micro-structural modeling with lower-bound shakedown theory, the proposed scheme delivers a robust and computationally efficient approach for evaluating the performance of PRMMCs under variable and extreme loading conditions. The results demonstrate its compelling ability to accurately predict shakedown limits, proving its value as a tool for the support of safe and optimized design of next-generation, high-performance structural materials.

References

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