A deviatoric strain energy function for inflated elastic membranes

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EXTENDED ABSTRACT

Inflated elastic membranes exhibit complex mechanical and kinematic responses that are not fully captured by existing constitutive models. In this work, we introduce a deviatoric strain energy function (SEF) specifically formulated to address this limitation. The model parameters are identified through bulge-test experiments conducted on two elastomers, providing pressure–deflection data together with measurements of transversal and radial stretches at the membrane pole. These observations enable a comprehensive validation, showing that the proposed SEF reproduces the inflation behaviour of circular membranes more accurately than current models. The formulation features a simple polynomial structure, offers a clear physical interpretation of the influence of each term on the pressure response, and includes a well-defined fitting strategy. A complete constitutive model for inflated membranes is obtained when coupling the proposed SEF with a volumetric part previously developed by the authors. This work provides both novel experimental insights and theoretical improvements to the mechanics of inflated membranes and serves as foundation for the study of more complex problems.

EXPERIMENTAL TESTS

We performed bulge tests on four specimens of EPDM and NR elastomers [1]. The bulge tests were carried out using a custom pressure device equipped with pressure and laser sensors to record the pressure vs. pole's deflection curve. In addition, during the first test for both materials a camera placed at the top recorded the test to reconstruct the radial stretch profile at the pole. For the three remaining tests, the camera was replaced by a second laser to measure the transversal stretch at the pole. Details of the setup are shown in Fig.1. The resulting pressure–deflection curves showed highly repeatable behavior for both materials and their average was assumed as experimental data. The experimental results are shown in dot lines in Fig. 2.

HYPERELASTIC MODEL AND FITTING

We tested the performance of the most popular deviatoric SEFs on our experimental data, highlighting the limitations of current models to comprehensively describe the pressure-curve and the pole's stretches of our materials [1]. For this reason, we developed a novel deviatoric model specifically calibrated to simulate the experimental data on inflated membranes.

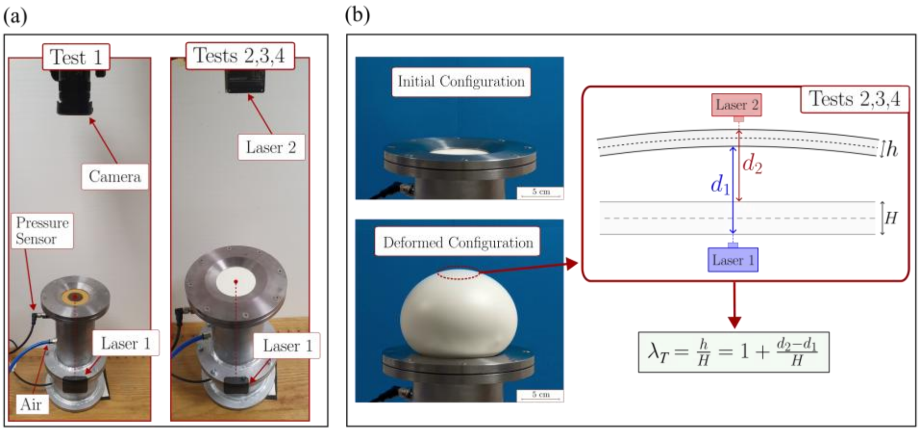

Figure 1: Setup of bulge tests. a): Experimental setup with lasers and cameras. b) Details on the transversal stretch measurement.

The proposed SEF reads:

$$W_d\left(\bar{I}_1, \bar{I}_2\right) = A\left(\bar{I}_1 - 3\right) + B\left(\bar{I}_2 - 3\right)\phi_s\left(\bar{I}_2\right) + C\left(\bar{I}_1 - 3\right)^4$$

Where $\bar{I}_1, \bar{I}_2$ are the isochoric invariants of strain and $\phi_s(\bar{I}_2)$ is a function governing softening for the term $B(I_2 - 3)$. The complete SEF is obtained combining the proposed function with the volumetric SEF developed in [2]. The results of the fitting to the experimental data is shown in Fig. 2. Not only the proposed model matches the pressure curve, but it provides an accurate description of the deformation at the pole for the tested materials. In addition, the proposed function has some advantages: 1) it consists in a simple polynomial expression, 2) the terms contributing to the SEF act on different parts of the pressure curve, enabling an easy interpretation of their role. These features help significantly the calibration of the model parameters and their control.

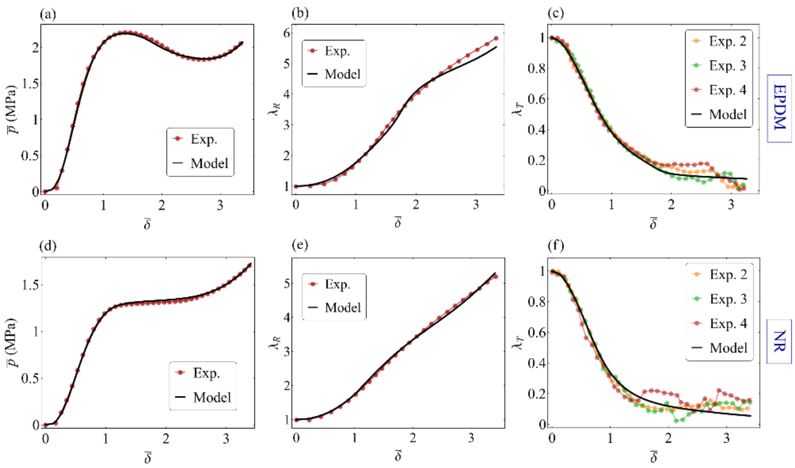

Figure 2: Fitting of the proposed SEF to the experimental data.

REFERENCES

[1] Sirotti, S., Pelliciari, M., Aloisio, A., & Tarantino, A. M. (2025). A strain energy function for the inflation of hyperelastic membranes. *Mechanics of Materials*, 105442.
[2] Pelliciari, M., Sirotti, S., & Tarantino, A. M. (2023). A strain energy function for large deformations of compressible elastomers. *Journal of the Mechanics and Physics of Solids*, *176*, 105308.